Ambient seismic frequency-domain gradiometry

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ABSTRACT

Seismic gradiometry has been used to estimate surface-wave phase velocity using spatial and temporal wavefield gradients. However, current formulations show limited depth sensitivity (de Ridder and Curtis, 2017). One ad hoc depth control approach applies narrow bandpass filtering to isolate specific frequency ranges before applying gradiometric processing on uncorrelated ambient seismic data (de Ridder and Biondi, 2015). Here, we propose an alternative: adapting gradiometry to frequency-domain virtual shot gather (VSG) data generated by seismic interferometry from ocean bottom node (OBN) recordings. When applied to large OBN arrays with sufficient spatial sampling and continuously recording over weeks-to-months period, VSGs can recover long-wavelength (tens of kilometers) Green's function wavefield information at enhanced signal-to-noise ratios (SNR). We explicitly extract low-frequency (sub-1 Hz) Scholte-wave energy and apply frequency-domain gradiometry (FDG) directly to VSG data (i.e., VSG-FDG), which obviates the need for narrow bandpass filtering and stacking over all time samples. To enhance VSG-FDG slowness estimates, we stack over virtual shots to generate higher-quality surface-wave phase-slowness volumes s(x,y,ω). This analysis opens the possibility of applying 1-D surface-wave inversion to s(x,y,ω) to generate a pseudo-3D shear-wave depth slowness model s(x,y,z) that can serve as long-wavelength input to elastic full waveform inversion (E-FWI).

METHOD

We extend existing time-domain gradiometry methods to VSG-FDG. We begin with the Helmholtz equation:

$$\left[\nabla^2 + \omega^2 s^2(\mathbf{x},\omega)\right] \hat{U}(\mathbf{x},\omega) = 0 \tag{1}$$

where $\hat{U}(\mathbf{x},\omega)$ is the Fourier transformed frequency domain wavefield at receiver location **x** and frequency ω. Next, we define the frequency-dependent virtual source cross-correlation function, $\hat{V}(\mathbf{x},\mathbf{x}_s,\omega)$ as:

$$\hat{V}(\mathbf{x},\mathbf{x}_S,\omega) = \hat{U}^*(\mathbf{x}_S,\omega)\hat{U}(\mathbf{x},\omega) \tag{2}$$

where $\mathbf{x}_s$ is the location of the virtual source, and $\tilde{U}^*$ is the complex conjugate of the Fourier transformed wavefield. We then multiply equation 1 with the complex conjugate $\tilde{U}^*(\mathbf{x}_s,\omega)$ i.e. cross-correlation:

$$\hat{U}^*(\mathbf{x}_S,\omega)\nabla^2\hat{U}(\mathbf{x},\omega) - (i\omega)^2 s^2(\mathbf{x},\omega)\hat{U}^*(\mathbf{x}_S,\omega)\hat{U}(\mathbf{x},\omega) = 0 \tag{3}$$

Combining equations 2 and 3 simplifies it to:

$$\nabla^2\hat{V}(\mathbf{x},\mathbf{x}_S,\omega) - (i\omega)^2 s^2(\mathbf{x},\omega)\hat{V}(\mathbf{x},\mathbf{x}_S,\omega) = 0 \tag{4}$$

Performing least-squares regression on equation 4 yields the slowness squared, $\hat{s}^2$ (i.e., "sloth") estimate:

$$\hat{s}^2(\mathbf{x},\mathbf{x}_\mathbf{S},\omega) = \frac{\hat{V}^*(\mathbf{x},\mathbf{x}_\mathbf{S},\omega)\nabla^2\hat{V}(\mathbf{x},\mathbf{x}_\mathbf{S},\omega)}{(i\omega)^2\hat{V}^*(\mathbf{x},\mathbf{x}_\mathbf{S},\omega)\hat{V}(\mathbf{x},\mathbf{x}_\mathbf{S},\omega)} \tag{5}$$

Equation 5 holds irrespectively of how well the cross-correlation function approximates the homogeneous Green's function given the ambient noise requirements for seismic interferometry. The mean surface-wave slowness field can now be estimated by stacking and normalizing over N_s total VSGs locations

$$s^2(\mathbf{x},\omega) = \frac{1}{N_S}\sum_{\mathbf{x}_\mathbf{S}} \hat{s}^2(\mathbf{x},\mathbf{x}_\mathbf{S},\omega) \tag{6}$$

which results in a function of 3-D space and frequency that facilitates local 1-D surface-wave inversion for generating a pseudo-3-D S-wave velocity model.

Before applying the frequency-domain gradiometry approach to an ambient VSG data volume, the correlated VSG data must be filtered within the frequency band of individual surface-wave modes (e.g., the fundamental Scholte wave mode). We than apply a masking operator using a toroidal filter to isolate observed surface-wave energy located in the inner and outer toroidal regions and remove other types of observed wave phenomena (including scattered). Additional processing includes using a smoothed mask to suppress edge artifacts on phase slowness maps after calculated spatial Laplacian derivatives and applying a median filter to reduce noise while preserving features.

We apply the introduced VSG-FDG method to a large OBN dataset from the Gulf of Mexico. The results show strong correlation between surface-wave slowness estimates and features in a high-resolution FWI P-wave velocity model derived from active-source seismic data. Depth sensitivity kernel analysis reveals structures to depth estimates of at least 4 km and a surface-wave slowness decreasing with depth trend that correlates with the increasing FWI P-wave velocity model gradients. These observations support the assertion of waveform sensitivity to large-scale salt structures and deeper velocity model structure. This work is a key step toward generating a long-wavelength pseudo-3-D S-wave velocity model, which could serve as a sufficient starting model for 3-D E-FWI.

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